Complete the square. $\left(\frac{b}{2a}\right)^2$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

6-5 The Quadratic Formula and the Discriminant

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex 1

$3x^2 + x - 1 = 0$

$a = 3$
$b = 1$
$c = -1$

$x = \frac{-1 \pm \sqrt{1 - 4(3)(-1)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{13}}{6}$

ex 2

$5x^2 + 8 = -12x$

$5x^2 + 12x + 8 = 0$

$a = 5$
$b = 12$
$c = 8$

$x = \frac{-12 \pm \sqrt{12^2 - 4(5)(8)}}{2(5)}$

$\frac{-12 \pm \sqrt{-16}}{10}$

$\frac{-12 \pm 4i}{10}$

$x = \frac{-6 \pm 2i}{5}$

The Discriminant

$$D = b^2 - 4ac$$

Determines the nature of the roots.

Three Cases

I. $D > 0$ 2 $\mathbb{R}$ Roots

II. $D = 0$ 1 $\mathbb{R}$ root (double)

III. $D < 0$ 2 imaginary roots

Ex
$x^2 - 8x + 5 = 0$

Determine the nature of the roots

$D = b^2 - 4ac$
$= 64 - 4(1)(5)$
$= 44$

2 $\mathbb{R}$ roots, irrational

Also able to determine if the roots are rational or irrational.

Rational
a, b, & c must be rational and D must be a perfect square (Real)

ex

$x^2 + 10x + 25 = 0$

$D = 100 - 4(1)(25)$
$= 0$

1 double ℝ root, rational

ex

$x^2 - 4x + 13 = 0$

$D = 16 - 4(1)(13)$
$16 - 52 = -36$

2 imaginary roots

Determine as much as you can about the roots:

1. $y^2 - 3y - 1 = 0$

2. $3a^2 - 10a = -11$

3. $5x^2 + (2\sqrt{10})x + 2 = 0$

4. $3b^2 = 14b + 24$

$D = (2\sqrt{10})^2 - 4(5)(2)$
$40 \quad -40$
$D = 0$

Double ℝ root, irrational

Find the value for k such that there are 2 imaginary roots

ex

$5x^2 - 2x + k = 0$ $\quad D < 0$

$4 - 4(5)k < 0$
$-20k < -4$
$k > \frac{1}{5}$ $\quad k > \frac{1}{5}$

ℝ

Find the value for k such that there is a double root

ex

$3x^2 + 2x + k = 0$

$4 - 4(3)k = 0$
$-12k = -4$
$k = \frac{1}{3}$

p318
15-25 odd and 45a
(not #21)